

Ghost Cosmology: Exact Solutions, Transitions Between Standard Cosmologies and Ghost Dark Energy/Matter Evolution

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(Dated: February 1, 2008)

Abstract

The recently proposed infrared modification of gravity through the introduction of a ghost scalar field results in a number of interesting cosmological and phenomenological implications. In this paper, we derive the exact cosmological solutions for a number of scenarios where at late stages, the ghost behaves like dark matter, or dark energy. The full solutions give valuable information about the non-linear regime beyond the asymptotic first order analysis presented in the literature. The generic feature is that these ghost cosmologies give rise to smooth transitions between radiation dominated phases (or more general power-law expansions) at early epochs and ghost dark matter resp. ghost dark energy dominated late epochs. The current age of our universe places us right at the non-linear transition phase. By studying the evolution backwards in time, we find that the dominance of the ghost over ordinary baryonic matter and radiative contributions persists back to the earliest times such that the Friedmann-Robertson-Walker geometry is dictated to a good approximation by the ghost alone. We also find that the Jeans instability occurs in the ghost dark energy scenario at late times, while it is absent in the ghost dark matter scenario.

PACS numbers: 98.80.Cq, 04.20.Jb

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I. INTRODUCTION

An interesting proposal to explain dark matter or dark energy by an infrared modification of standard gravity was made recently in [1] (there is a rather long history of earlier attempts at infrared modifications of gravity, see e.g. [2]). The key idea is that gravity at the present epoch might be in a Higgs phase. This is achieved by the introduction of an additional ghost scalar field, ϕ , with the wrong-sign kinetic term. If the scalar has a vanishing vacuum expectation value (vev), one would obviously obtain an unstable vacuum since the kinetic energy would be unbounded from below. However, a stable vacuum can emerge if one allows for a more general kinetic term. The ghost scalar might then condense in a vacuum with non-trivial constant velocity in field space

$$\phi(t) = \pm\sqrt{C}t. \quad (1)$$

Notice that this vacuum breaks Lorentz invariance spontaneously by breaking time translation invariance. The ghost's kinetic term is described by a kinetic function $P(X)$ where [15]

$$X = -\partial_\mu\phi\partial^\mu\phi \quad (2)$$

is (up to a factor two) the standard kinetic term. The interesting observation is then that, once the ghost has condensed, it assumes the equation of state of either matter or vacuum energy. Consequently it represents an interesting candidate for both dark matter and dark energy [1].

To study fully the cosmological consequences of the ghost condensation proposal one first needs to derive the corresponding cosmological solutions. Though asymptotic analysis at late stages, close to the condensation point, does already provide important insights, it cannot address the full history of the ghost's evolution. In particular one might ask what happens to the ghost's mass density when we adjust it, according to the proposal of [1], at the current epoch with the observed dark matter/energy density. Will the ghost still dominate at earlier times or become negligible as compared to ordinary matter resp. radiation? How is the scale-factor of the universe affected by the ghost at earlier times? Moreover, with the current age of the universe, as given by its Hubble time $H_0^{-1} = (10^{-33}\text{eV})^{-1}$, has the ghost already had sufficient time to condense, or are we still far away from this critical point. Moreover, one would like to know when the asymptotic behavior sets in and can be trusted as an approximate solution.

Our goal in this paper is to provide these ghost cosmological solutions for a number of cases, as a starting point for more detailed cosmological and astrophysical investigations of the ghost condensation proposal. We will see that a ghost cosmology leads generically to a universe which evolves from a fractional power-law expansion (for the simplest model with quadratic $P(X)$ one finds a $t^{1/2}$ radiative phase; for models based on cubic, quartic or higher power $P(X)$ more exotic fractional power time dependences arise) to a late time matter or dark energy dominated phase. A ghost cosmology defined through a particular $P(X)$ will therefore smoothly interpolate between two types of standard cosmologies. This in itself seems to be of technical interest, given that normally when going from one epoch to the next, one simply glues standard cosmologies together in a continuous but non-smooth fashion. Furthermore, we will see that at the current epoch, if the ghost indeed accounts for a sizeable contribution to dark matter/energy, neither of the asymptotic late or early time limiting

cosmological solutions applies. Instead, we find ourselves right in the highly non-linear transitory stage between early and late time asymptotics. Hence the ghost condensation process, which is shown to endure for an infinite amount of time, is far from being finished. Via the ghost cosmological solutions the ghost's mass density will depend in a non-trivial way on cosmic time. In particular, it will turn out that if the current ghost's mass density is of the size of the presently observed dark matter/energy density, then the ghost will always have dominated other contributions like ordinary matter or radiation in the past and will continue to do so in the future. Thus as a good approximation one can neglect ordinary matter and radiation at other epochs as well and get a sufficiently accurate cosmological description by just considering the ghost-gravity system alone. Our thus derived ghost cosmologies are therefore relevant not only at the current epoch but also at earlier and later epochs. One interesting aspect of these ghost cosmological solutions concerns the coincidence problem. In general, since the ghost's contribution to the dark energy density does not stay constant in time one would expect not only the dark matter density but now also the ghost's dark energy density to increase towards earlier times. This should at least lead to some alleviation of the coincidence problem and would be interesting to study in future work.

Finally, it would be nice to embed the ghost condensation proposal into M-theory. The ghost, which appears only derivatively coupled, satisfies a shift symmetry and could therefore easily be identified as an axion. The hope would be that M-theory can fix the magnitude of the ghost dark matter or energy which in the effective field theory framework needs to be adjusted to the observed values. Moreover M-theory predicts dark matter/dark energy on its own (see e.g. [4]) and it would be interesting to see the interplay of these contributions with the ghost contribution. For other interesting implications of the ghost condensate proposal or related subjects, see the references in [5].

Without further a due, we will start the next section by presenting the general framework for cosmology with the ghost scalar. Section III will be devoted to the full cosmological solution related to the appearance of ghost dark matter at late epochs. Here, as we have alluded to already, we find a transition from a radiation dominated early epoch to a ghost dark matter dominated late epoch. Section IV gives the cosmological solution for $P(X) = X^n$ which arises in any Taylor-expansion of a more complicated kinetic function and is in particular relevant at early times. The outcome is a simple fractional power-law time dependence. Section V derives the cosmological solutions which give rise to a late time (ghost) dark energy accelerated expansion. The general result is that the full solution interpolates between an early epoch radiation dominated phase and the mentioned (ghost) dark energy dominated late epoch. In Section VI, we discuss the possibility of a transition between an early period with large vacuum energy, suitable for instance to sustain inflation, and a late period with a smaller amount of dark energy. Section VII includes ordinary baryonic matter and radiation in addition to the ghost. We study their evolution and find that if the ghost accounts for all of dark matter resp. dark energy observed today that the dominance of the ghost will exist not only today but persists back to much earlier times. For the evolution of the scale-factor of the Friedmann-Robertson-Walker (FRW) geometry one can therefore neglect other sources besides the ghost to a very good approximation. The final section VIII analyzes the potential Jeans instability of our cosmological solutions.

II. GHOST COSMOLOGY

In this section, we will present the general framework describing the cosmological evolution of a FRW spacetime in the framework of general relativity coupled to a ghost scalar. In later sections, we will then apply this formalism to concrete cases covering late time generation of ghost dark matter or dark energy.

A. The Cosmological Equations

Following the proposal of [1], one replaces the usual kinetic term X for the ghost scalar ϕ through some more complicated function $P(X)$ leading to a ghost gravity theory defined by an action

$$S = \int d^4x \sqrt{-g} \left(\frac{R}{16\pi G} + M^4 P(X) \right) . \quad (3)$$

Notice that it is not necessary to add a cosmological constant term separately by hand as it can be absorbed into a constant added to $P(X)$. We will elaborate more on this issue later in section V. The mass scale M is not determined *a priori* by the theory itself but should be regarded as a free parameter which needs to be fixed by phenomenological considerations. The range of M becomes thus confined to [1]

$$1\text{meV} \leq M \leq 10\text{MeV} \quad (4)$$

where the upper bound can possibly even be relaxed to the GeV regime [6]. By variation of this action w.r.t. the metric and ϕ , we obtain the Einstein equation

$$G_{\mu\nu} = 8\pi G T_{\phi,\mu\nu} \quad (5)$$

with ghost energy-momentum tensor

$$T_{\phi,\mu\nu} = M^4 (P(X) g_{\mu\nu} + 2P'(X) \partial_\mu \phi \partial_\nu \phi) \quad (6)$$

and the equation of motion for ϕ

$$\partial_\mu (\sqrt{-g} P'(X) \partial^\mu \phi) = 0 . \quad (7)$$

For the ghost condensate proposal, it is essential that $P(X)$ exhibits a minimum at some positive value C such that X becomes timelike at this value and consequently ϕ acquires a non-vanishing time-dependent vev (1) in this vacuum state. A stability analysis of fluctuations around the condensate point implies the constraints [1]

$$P'(C) \geq 0 , \quad P'(C) + 2CP''(C) \geq 0 . \quad (8)$$

Besides these requirements, $P(X)$ is *a priori* arbitrary. Though it will soon turn out that the Friedmann equation (for the case with flat spatial sections studied in this paper) imposes a further consistency constraint, it is typically the case that this additional constraint is, for the cases of $P(X)$ studied in this paper, satisfied wherever (8) holds true.

The ghost condensate breaks time translation invariance and the shift symmetry in ϕ down to a diagonal subgroup [1] while leaving all other spatial symmetries like e.g. rotation

invariance intact. A spatially homogeneous and isotropic universe is therefore compatible with the idea of a ghost condensate and will be adopted throughout this paper. The corresponding geometry is described by an FRW metric where our focus is on the case with flat 3d sections

$$ds^2 = -dt^2 + a^2(t) (dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)) . \quad (9)$$

In compliance with the homogeneity and isotropy assumption, the ghost scalar cannot depend on the spatial coordinates but only on time

$$\phi = \phi(t) . \quad (10)$$

We therefore see that in the FRW background X depends only on the ghost's time derivative

$$X = \dot{\phi}^2 . \quad (11)$$

Consequently the equation of motion for ϕ simplifies to

$$\frac{d}{dt}(a^3(t)\dot{\phi}P'(X)) = 0 , \quad (12)$$

which is solved by

$$a^3(t)\dot{\phi}P'(X) = \text{const} . \quad (13)$$

If at late times we want to have an ever increasing scale-factor $a \rightarrow \infty$ then either $\dot{\phi}$ or $P'(X)$ have to vanish. The former choice, which would amount to redshifting the ghost velocity to zero, does not take us to the ghost condensate vacuum state and will not be of interest to us here. We will therefore adopt the second boundary condition, namely that asymptotically,

$$X \rightarrow C \quad \text{with} \quad P'(C) = 0 \quad \text{as} \quad t \rightarrow \infty , \quad (14)$$

which describes the condensation of the ghost $\phi(t) \rightarrow \pm\sqrt{C}t$.

This boundary condition implies that the condensation process takes place in an infinite amount of time. In view of the fact that the condensation point $X = C$ for the simplest quadratic kinetic function $P(X) = \frac{1}{2}(X - C)^2$ marks the border between stable and unstable states [1] this is what we want. Otherwise, one would have to be concerned about ‘overshooting’ into the unstable regime when ϕ condenses. Nevertheless, one might also wonder whether it might be possible for the condensation process to finish in a finite amount of time. If we assume this to be true then we learn from Eq. (13) that $a(t_c) = \infty$ would have to hold at finite time t_c indicating that the solution possesses a singularity which can be reached in finite time and is geodesically incomplete. We will discard such solutions and are hence forced to assume that the condensation process terminates at $t_c = \infty$.

From the Einstein equation, two independent equations are obtained. Thus, it might seem that the problem might be overdetermined: three equations (two from the Einstein equation plus the ghost equation of motion) for just two unknown functions (the scale-factor $a(t)$ and the ghost $\phi(t)$). This is, however, not true. To this end, let us note that

$$\nabla^\mu T_{\mu\nu}^\phi = -\delta_\nu^0 \frac{d}{dt}(a^3\sqrt{X}P') , \quad (15)$$

where we have used that $\dot{P} = P'\dot{X}$. Hence, local energy conservation is equivalent to the equation of motion for the ghost given earlier in Eq. (12).

$$\nabla^\mu T_{\mu\nu}^\phi = 0 \quad \Leftrightarrow \quad \text{EOM for } \phi. \quad (16)$$

Since local energy conservation is already implied by the Einstein equation, the same is true for the ghost equation of motion. We are therefore left with just two independent equations coming from the Einstein equation for the two unknown functions $a(t)$, $X(t)$. By noticing that the ghost depends only on the time coordinate, we can always replace its time derivative by \sqrt{X} and thus work exclusively with the more convenient variable X . This would no longer be possible if we allow for an inhomogeneous or anisotropic universe as in these cases, the ghost would depend on more than just one coordinate.

Having dealt with the scalar equation of motion, let us now consider the equations for the metric. For the FRW background, the Einstein equation leads to two independent equations, the *Friedmann equation*

$$H^2 = \frac{m^2}{3}(2XP'(X) - P(X)) \quad (17)$$

and the evolution equation

$$H^2 + 2\frac{\ddot{a}}{a} = -m^2 P(X) \quad (18)$$

where, as usual, the Hubble parameter H denotes \dot{a}/a and we have defined the mass-parameter m through

$$m = \sqrt{8\pi G}M^2 = \frac{M^2}{M_{Pl}}, \quad (19)$$

with M_{Pl} the reduced Planck mass. Like the “see-saw” relation of Grand Unified Theories employed to explain the smallness of the neutrino masses [8], we have here a similar “see-saw” relation where M is the geometric mean of m and M_{Pl} . With a range of M between 1meV and 1GeV the parameter m lies between the extremely small energies

$$4 \times 10^{-34}\text{eV} < m < 4 \times 10^{-10}\text{eV}. \quad (20)$$

Notice that the lower bound corresponds to the current Hubble scale which is $H_0 \simeq 10^{-33}\text{eV}$.

Combining the evolution and the Friedmann equation to eliminate H from the former yields the *acceleration equation*

$$\frac{\ddot{a}}{a} = -\frac{m^2}{3}(XP'(X) + P(X)) \quad (21)$$

which is more convenient to use than the evolution equation.

B. The General Formal Solution to the Cosmological Equations

The Friedmann equation (for the 3d flat case with $k = 0$ considered here), due to the positivity of H^2 , imposes another consistency constraint

$$2XP'(X) \geq P(X), \quad \forall X(t) \quad (22)$$

on any candidate ghost kinetic function $P(X)$. We will see shortly that this constraint is equivalent to demanding a non-negative gravitating mass density ρ_ϕ coming from the ghost. It should, however, be noted that further sources of positive mass densities or a negative 3d curvature would contribute positively to H^2 , thereby relaxing the above constraint somewhat. Let us now proceed by formally solving the Friedmann and the acceleration equation.

First, one observes that the Friedmann equation, a first order differential equation for $a(t)$, can be formally solved in terms of X through ($a_i = a(t_i)$)

$$a(t) = a_i e^{f(t, t_i)}, \quad f(t, t_i) = \pm \frac{m}{\sqrt{3}} \int_{t_i}^t dt' \sqrt{2X(t')P'(X(t')) - P(X(t'))} \quad (23)$$

with t_i an initial reference time. Now, we can plug this solution into the lhs of the acceleration equation to eliminate the scale-factor and obtain the following first order differential equation for $X(t)$

$$2(P'(X))X + \dot{P} = \mp 2\sqrt{3}mXP'(X)\sqrt{2XP'(X) - P(X)}, \quad (24)$$

which can also be written more compactly as

$$\frac{d}{dt} \ln |(P'(X))^2 X| = \mp 2\sqrt{3}m\sqrt{2XP'(X) - P(X)}. \quad (25)$$

This is then the equation, which when supplemented with the boundary condition (14), determines $X(t)$. Assuming that $X(t)$ has been found by solving (25) we can then evaluate explicitly the function $f(t, t_i)$ and thus determine the scale-factor from (23). Indeed, with help of (25) we can show for general $P(X)$ that

$$f(t, t_i) = -\frac{1}{6} \int_{t_i}^t dt' \frac{d}{dt'} \ln |X(P'(X))^2| = \ln \left| \left(\frac{X_i(P'(X_i))^2}{X(t)(P'(X(t)))^2} \right)^{1/6} \right|, \quad (26)$$

which gives us for the scale-factor

$$a(t) = a_i \left| \frac{X_i(P'(X_i))^2}{X(t)(P'(X(t)))^2} \right|^{1/6}. \quad (27)$$

As can be easily verified, this solution for $a(t)$ satisfies (13) and therefore is a solution to the ghost equation of motion. This is in accordance with what we said earlier about the redundancy of the ghost equation of motion as it is already implied by the Einstein equation.

In the following, we will solve (25) for three classes of functions $P(X)$ and study the corresponding ghost cosmologies. But before doing so, let us briefly return to the energy-momentum tensor (6). In general the diagonal components of the energy-momentum tensor represent the mass density and pressure, $T_{\mu\nu} = (\rho, pg_{rr}, pg_{\theta\theta}, pg_{\phi\phi})$. By comparison to (6) one may therefore also infer a pressure and a mass density associated with the ghost field

$$\rho_\phi = M^4(2XP'(X) - P(X)), \quad p_\phi = M^4P(X). \quad (28)$$

These quantities will turn out to be very useful in understanding various limits of our cosmological solutions. The constraint (22) implies that ρ_ϕ can never become negative.

III. GHOST DARK MATTER CASE: TRANSITION FROM RADIATION- TO MATTER-DOMINATED UNIVERSE WITH $P(X) = \frac{1}{2}(X - C)^2$

Let us now apply the formalism of the previous section to some concrete models. The simplest choice for $P(X)$ letting the ghost condense at some finite positive value $X = C$ is given by the parabola

$$P(X) = \frac{1}{2}(X - C)^2 . \quad (29)$$

The model defined by this kinetic function will be relevant for the question of ghost dark matter at the present epoch as we will see. Both the stability criterion (8) and the constraint (22) require that we work in the regime where $X \geq C > 0$. Notice that in the excluded regime of negative X , it would be required by (11) that we work with a complex ghost field while for positive X , it is consistent to work with a real field.

The Friedmann and acceleration equation become

$$H^2 = \frac{m^2}{6}(X - C)(3X + C) \quad (30)$$

$$\frac{\ddot{a}}{a} = -\frac{m^2}{6}(X - C)(3X - C) , \quad (31)$$

and it follows from the acceleration equation that any cosmological solution must describe a *decelerated expansion* for which

$$\ddot{a} \leq 0 . \quad (32)$$

where equality is attained when the ghost reaches the condensation point $X \rightarrow C$ at late times.

The differential equation (25) determining $X(t)$ can be written most conveniently as

$$\dot{X} = \mp \sqrt{6}m \frac{X(X - C)^{3/2}(3X + C)^{1/2}}{3X - C} \quad (33)$$

and gets solved by the following implicit expression for $X(t)$

$$Q(Y) + 2 \arctan Q(Y) - Q_i - 2 \arctan Q_i = \pm \sqrt{6}Cm(t - t_i) \quad (34)$$

where

$$Q(Y) = \sqrt{\frac{3Y(t) + 1}{Y(t) - 1}} , \quad Y(t) = \frac{X(t)}{C} . \quad (35)$$

and $Q_i = Q(t_i)$ denotes the value at initial time t_i . Subsequently, we will choose the solution with positive sign leading to a time flow in positive direction while the ghost scalar condenses. To determine its value we impose the following initial boundary condition

$$X(t_i) \gg C, \quad Y(t_i) \gg 1 \quad \text{as} \quad t \rightarrow t_i = 0 , \quad (36)$$

which fixes the integration constant with

$$Q_i = \sqrt{3} . \quad (37)$$

Notice that at late times where the boundary condition must be (14), it cannot be determined as it drops out in this limit. This solution for $X(t)$ maps the time interval $t_i = 0 \leq t \leq \infty$ faithfully into the X interval $\infty \geq X(t) \geq C$. *The ghost condensation process is therefore seen to take an infinite amount of time.* Therefore, once the cosmological evolution starts within the regime where $X > C$, we don't have to worry that we might be driven towards the unstable regime where $X < C$ in finite time. The time-dependence of the FRW scale-factor follows from (27) as

$$a(t) \propto \frac{1}{X(t)^{1/6}(X(t) - C)^{1/3}} \propto \frac{1}{Y(t)^{1/6}(Y(t) - 1)^{1/3}} . \quad (38)$$

Before we discuss this solution, let us first look into its early and late time behavior. Close to the initial time where

$$t \rightarrow 0 , \quad Y \rightarrow \infty , \quad (39)$$

we can expand the solution for $Y(t)$ and $a(t)$ and find

$$Y(t) = \frac{1}{\sqrt{2Cmt}} \quad (40)$$

$$a(t) \propto (Cmt)^{1/2} . \quad (41)$$

Hence at early times, the ghost scalar behaves like radiation and we obtain the characteristic $a(t) \propto t^{1/2}$ behavior of a *radiation-dominated type universe*.

On the other hand at late times when the ghost has nearly reached its condensation point (Y approaches 1 from above)

$$t \rightarrow \infty , \quad Y \rightarrow 1 , \quad (42)$$

expansion of the solution leads to the asymptotic solution

$$Y(t) = 1 + \frac{2}{3(Cmt)^2} \quad (43)$$

$$a(t) \propto (Cmt)^{2/3} . \quad (44)$$

Thus, as expected, the ghost field behaves at late times like non-relativistic matter with the characteristic $a(t) \propto t^{2/3}$ behavior. It could therefore account for some or all of dark matter at the present epoch. The time evolution of this type of ghost dark matter will be investigated in section VII. *The full cosmological solution (34),(38) therefore describes a smooth transition from a radiation dominated early epoch to a ghost dark matter dominated late epoch.*

This transition from a radiation- to a matter-dominated universe can also be understood by inspection of the ghost's matter density ρ_ϕ and pressure p_ϕ at early and late times. For $t \rightarrow 0, X(t) \gg C$ the expressions (28), with $P(X)$ given in (29), yield

$$\rho_\phi \rightarrow 3p_\phi \quad (45)$$

while at late times towards the end of the condensation process, $t \rightarrow t_c = \infty, X \rightarrow C$ we obtain

$$\rho_\phi = 2M^4C(X - C) + \mathcal{O}((X - C)^2) \gg p_\phi = \mathcal{O}((X - C)^2) . \quad (46)$$

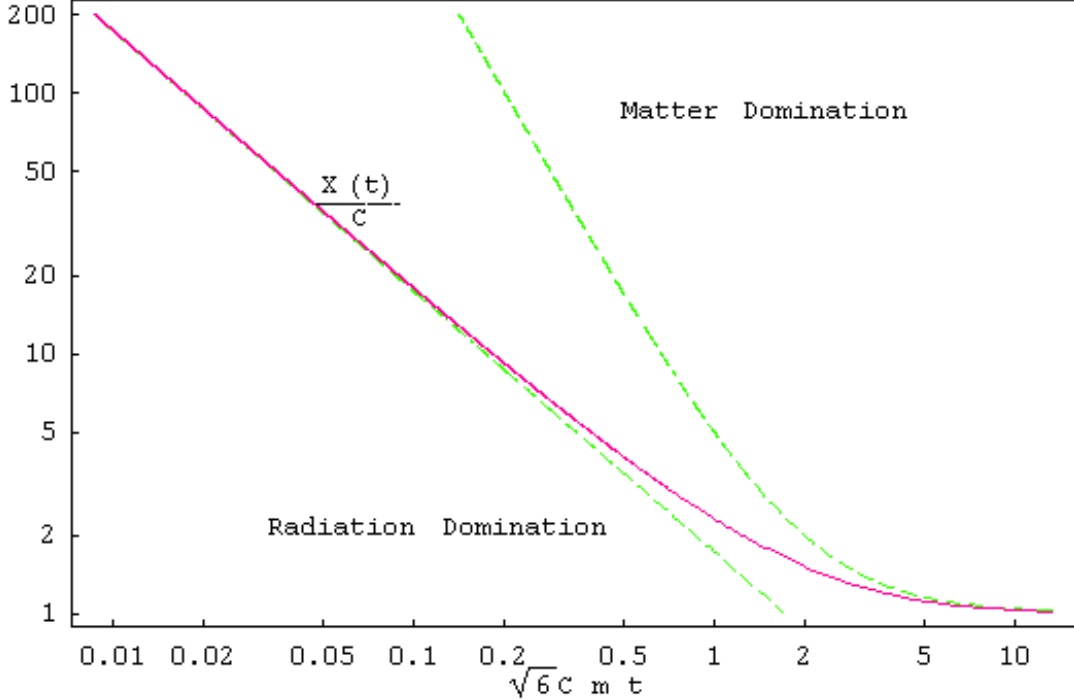


FIG. 1: Plot of the ghost field $X(t)$ (red middle curve) as a function of time for a theory based on $P(X) = \frac{1}{2}(X - C)^2$. It is clearly visible how $X(t)$ interpolates between the early time radiation dominated behavior (green lower dashed curve) and the late time ghost dark matter dominated behavior (green upper dashed curve). The transition occurs during the relatively short time interval $\sqrt{6}Cmt \sim 0.1 - 10$.

Hence, the ghost field assumes indeed the correct equation of state to behave as radiation at an early epoch and non-relativistic matter at a late epoch.

We will see later in section VII that with the current age of the universe $t_0 = 1/H_0$ we are led to $\sqrt{6}Cmt_0 \sim 1$. If we now plot the full solutions for $X(t)$ (Fig.1) and $a(t)$ (Fig.2) they show besides the discussed transition from a radiation to a matter dominated cosmology that we are *currently right in the transition period*. As an explanation of dark matter through the ghost, this is promising as we see from (46) that towards the very end of the condensation process ρ_ϕ will vanish and hence its contribution to non-relativistic matter as well.

IV. BEHAVIOR AT LARGE X AND THE GENERAL CASE: $P(X) = X^n$ AND $P(X) = (X - C)^n$

Exact cosmological solutions for more complicated $P(X)$ can become quite involved. It is therefore important to understand the general characteristics. Let us now concentrate on the cosmological evolution when $X \gg C$, i.e. at early times when the ghost is still far away from its condensation point. Obviously in this regime, any sufficiently smooth function $P(X)$ can be Taylor-expanded in terms of powers of X . Let us therefore study first the dynamics of just the simple power-law

$$P(X) = X^n, \quad (47)$$

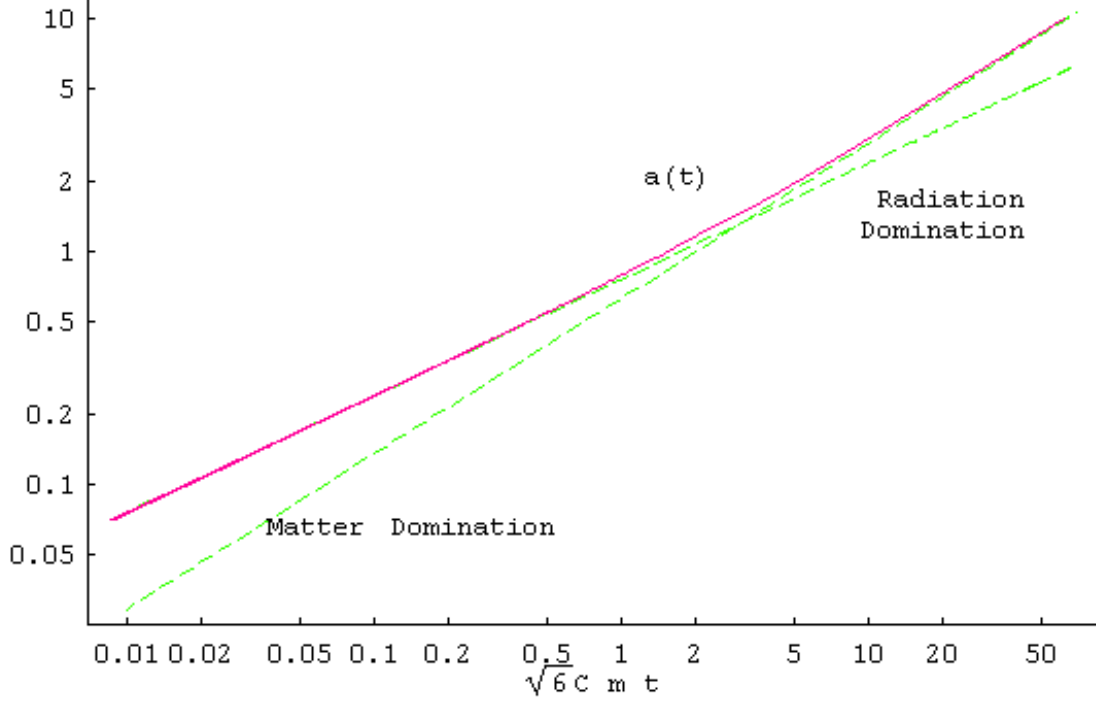


FIG. 2: Plot of the scale factor $a(t)$ as a function of time for a ghost cosmology with $P(X) = \frac{1}{2}(X - C)^2$. The scale factor (red middle curve) evolves from a $t^{1/2}$ radiation dominated behavior at early times (green upper dashed curve) to a $t^{2/3}$ ghost dark matter dominated behavior at late times (green lower dashed curve).

which would occur in the Taylor expansion of more complicated function $P(X)$. In this case the differential equation for $X(t)$ is easily solved through

$$X(t) = \left(\pm \frac{\sqrt{3}mn}{\sqrt{2n-1}}t + X_i^{-\frac{n}{2}} \right)^{-2/n}, \quad (48)$$

with initial value $X_i = X(t_i)$ at initial time $t_i = 0$. If we assume that initially

$$X_i \gg 1, \quad (49)$$

and take the solution with positive sign, then the result is a simple power dependence for $X(t)$ as well as for $a(t)$ (by plugging $X(t)$ into the general (27))

$$X(t) = \left(\frac{\sqrt{2n-1}}{\sqrt{3mn}} \right)^{2/n} \frac{1}{t^{2/n}} \quad (50)$$

$$a(t) \propto t^{\frac{2n-1}{3n}}. \quad (51)$$

Indeed, it is easy to understand how this time-dependence arises. Namely, for $P(X) = X^n$ we find that the mass density and pressure of the ghost obey

$$\rho_\phi = (2n - 1)p_\phi. \quad (52)$$

Expressed in terms of the usual equation of state $p = w\rho$, this means that $w_\phi = \frac{1}{2n-1}$. Given that for a FRW universe with flat spatial sections and constant equation of state parameter w one gets

$$a(t) \propto t^{\frac{2}{3(1+w)}} , \quad (53)$$

we see that with w_ϕ the solution (51) immediately follows. As a particular case, we recover the radiation dominated universe if $n = 2$.

The more complicated kinetic function

$$P(X) = (X - C)^n \quad (54)$$

which would, in contrast to the approximate $P(X) = X^n$, correctly describe a condensation process and generalize the simple parabolic $P(X)$ of the last section gives rise to the following equation for $Y(t)$

$$[(2n-1)Y - 1]\dot{Y} = \mp(2\sqrt{3}C^{\frac{n}{2}}m)Y(Y-1)^{\frac{n+1}{2}}[(2n-1)Y + 1]^{\frac{1}{2}} \quad (55)$$

which has a solution in terms of a combination of two Appell hypergeometric functions. Since this solution is rather involved, we will not present it here but confine ourselves to a discussion of the early and late time behavior. During the initial stages we have approximately a $P(X) = X^n$ behavior with the power-law expansion (51) while at late stages close to the ghost's condensation point when $X(t) \rightarrow C$ we find a ghost equation of state

$$\frac{p_\phi}{\rho_\phi} = (X - C)/2nC \rightarrow 0 \quad (56)$$

indicating a matter dominated period which once more might be used to describe dark matter. With the generalization (54) to arbitrary powers, we therefore recognize that *transitions from more general power-law cosmologies $a(t) \propto t^{(2n-1)/3n}$ at an early epoch to a late epoch ghost dark matter dominated $a(t) \propto t^{2/3}$ cosmology* are possible as well.

V. DARK ENERGY CASE: TRANSITION FROM RADIATION- TO DE SITTER UNIVERSE WITH $P(X) = \frac{1}{2}(X - C)^2 - D$

Let us next see how one can accommodate for a dark energy dominated de Sitter expansion phase at late times within the ghost gravity framework. Adding an explicit cosmological constant Λ to the ghost-gravity action leads to [16]

$$S = \int d^4x \sqrt{-g} \left(\frac{(R - 2\Lambda)}{16\pi G} + M^4 P(X) \right) . \quad (57)$$

From this action and also from the ensuing Einstein equation

$$G_{\mu\nu} = 8\pi G T_{\phi,\mu\nu} - \Lambda g_{\mu\nu} \quad (58)$$

with $T_{\phi,\mu\nu}$ as given in (6), it is obvious that a cosmological constant term can be completely absorbed into $P(X)$ simply by adding a constant

$$P(C) = -\frac{\Lambda}{m^2} \quad (59)$$

to the kinetic function, $P(X) \rightarrow P(X) + P(C)$. We can therefore still work with the simpler action (3) and use our formalism as presented in section II. Unlike the simple parabolic $P(X)$ case which we studied before in relation to dark matter which had $P(C) = 0$, we would now require a non-vanishing $P(C) \neq 0$ at the condensation point $X = C$ which is characterized furthermore through $P'(C) = 0$. This will lead to either de Sitter (dS) or anti-de Sitter (AdS) universes whenever such a critical point is reached

$$P(C) < 0 \quad \Rightarrow \quad \Lambda > 0 \quad \Rightarrow \quad \text{dS} \quad (60)$$

$$P(C) > 0 \quad \Rightarrow \quad \Lambda < 0 \quad \Rightarrow \quad \text{AdS} \quad (61)$$

The simplest theory exhibiting such a late time dS expansion arises by adding to our previous parabolic $P(X)$ a positive constant D

$$P(X) = \frac{1}{2}(X - C)^2 - D, \quad (62)$$

such that at the critical point $P(X = C) = -D$. Upon condensation the ghost then behaves like a positive cosmological constant

$$\Lambda = m^2 D. \quad (63)$$

The Friedmann and acceleration equations for this specific case become

$$H^2 = \frac{m^2}{3} \left(\frac{1}{2}(X - C)(3X + C) + D \right) \quad (64)$$

$$\frac{\ddot{a}}{a} = -\frac{m^2}{3} \left(\frac{1}{2}(X - C)(3X - C) - D \right). \quad (65)$$

It is obvious from the acceleration equation that *at early times* where $X \gg C$ the ensuing ghost cosmology undergoes a period of *deceleration*, $\ddot{a} < 0$, while *at late times* when the ghost condenses and we have $X \rightarrow C$ this will turn into a period of *acceleration*, $\ddot{a} > 0$. Working once more in the regime where $X \geq C$ will ensure that both constraints, (8) and (22), are satisfied such that the vacua considered are stable and the ghost mass density is always non-negative. This regime is also compatible with the boundary condition (14) at late times.

To find the corresponding cosmological evolution we have to solve the differential equation for $X(t)$ (25). To this end, it is most convenient to work with the rescaled variable $Y(t) = X(t)/C$ and to introduce the parameters

$$\tilde{m} = \pm \sqrt{6} m C, \quad d = \frac{2D}{C^2}. \quad (66)$$

The differential equation (25) then becomes

$$\dot{Y}(3Y - 1) = -\tilde{m}Y(Y - 1)\sqrt{(Y - 1)(3Y + 1) + d}. \quad (67)$$

Once a solution to this equation has been found, the scale-factor follows from (27) as

$$a(t) \propto \frac{1}{Y(t)^{1/6}(Y(t) - 1)^{1/3}}. \quad (68)$$

We will now present the solutions in different classes according to the value d which determines the late period cosmological constant.

A. $d > 1, d \neq \frac{4}{3}$

For the first case $d > 1, d \neq \frac{4}{3}$ which comprises the case with arbitrary large vacuum energy the differential equation yields the solution

$$\tilde{m}t = \pm \frac{1}{\sqrt{d-1}} \ln \left| \frac{U_{\pm}(t)}{U_{\pm,i}} \right| + \frac{2}{\sqrt{d}} \ln \left| \frac{V(t)}{V_i} \right|, \quad (69)$$

where

$$U_{\pm}(t) = \frac{2}{Y(t)} \left(\frac{\pm(d-1-Y(t))}{\sqrt{d-1}} + \sqrt{3Y^2(t) - 2Y(t) - 1 + d} \right), \quad (70)$$

$$V(t) = \frac{1}{(Y(t)-1)} \left(\frac{d+2(Y(t)-1)}{\sqrt{d}} + \sqrt{3Y^2(t) - 2Y(t) - 1 + d} \right) \quad (71)$$

with initial values $U_{\pm,i}, V_i$ at time $t_i = 0$ and either both plus signs or both minus signs have to be chosen. For $d = 4/3$ the function $U_{\pm}(t)$ would vanish identically in the relevant interval $1 \leq Y(t) < \infty$. We will therefore study this case separately and concentrate here on the case with $d \neq 4/3$. Moreover, we will focus on the solution with plus sign choice, i.e. U_+ .

The first thing to notice is that both $U_{\pm}(t)$ and $V(t)$ have no zero in the interval $1 \leq Y(t) < \infty$ and therefore do not change sign. Moreover, both $|U_+(Y(t))|$ and $V(Y(t))$ decrease monotonically with Y over the interval $1 \leq Y < \infty$ and cover the range

$$\infty \geq V(Y) \geq \frac{2}{\sqrt{d}} + \sqrt{3}, \quad 2 \left| \frac{d-2}{\sqrt{d-1}} + \sqrt{d} \right| \geq |U_+(Y)| \geq 2 \left| \frac{1}{\sqrt{d-1}} - \sqrt{3} \right|. \quad (72)$$

We therefore learn that $|\tilde{m}|t$ decreases monotonically with Y over the interval $1 \leq Y < \infty$. Hence, time increases if the *universe evolves from large to small Y values*. So if we let the universe start off at initial time $t_i = 0$ at some value $Y_i > 1$ and condense at later time t_c at value $Y(t_c) = 1$, then this evolution is described by

$$|\tilde{m}|t = \frac{1}{\sqrt{d-1}} \ln \left| \frac{U_+(t)}{U_{+,i}} \right| + \frac{2}{\sqrt{d}} \ln \left| \frac{V(t)}{V_i} \right| \quad (73)$$

By inverting this implicit solution one obtains the scale-factor via (27). Notice that at the condensation point t goes to infinity, thus

$$t_c = \infty \quad (74)$$

and we see again that *it takes an infinite amount of time for the ghost to condense* which is why we do not have to worry about entering the unstable regime where X is smaller than C .

B. $d = \frac{4}{3}$

For the special case with $d = \frac{4}{3}$ which we have left out in the previous analysis one finds the simple explicit solution

$$Y(t) = \left(1 - e^{-\frac{\tilde{m}(t-t_i)}{\sqrt{3}}} \left(1 - \frac{1}{Y_i} \right) \right)^{-1} \quad (75)$$

with value $Y_i > 1$ at initial time t_i . To have a solution in the stable regime $Y \geq 1$ we choose the positive sign for \tilde{m} . Again, we see that *the condensation process takes an infinite amount of time* until it reaches $Y(t_c = \infty) = 1$. With $t_i = 0$ the ensuing scale factor becomes

$$a(t) \propto e^{\frac{|\tilde{m}|t}{3\sqrt{3}}} \sqrt{1 - e^{-\frac{|\tilde{m}|t}{\sqrt{3}}} \left(1 - \frac{1}{Y_i}\right)}. \quad (76)$$

C. $d = 1$

For the case with $d = 1$ the solution reads

$$\tilde{m}t = 2 \ln \left| \left(\frac{W(t) + 1}{W_i + 1} \right) \left(\frac{W_i - 1}{W(t) - 1} \right) \right| - (W(t) - W_i), \quad (77)$$

where

$$W(t) = \sqrt{3} \left(\frac{\sqrt{1 + 3(3Y(t)^2 - 2Y(t))} - 1}{\sqrt{1 + 3(3Y(t)^2 - 2Y(t))} + 1} \right)^{1/2} \quad (78)$$

and initially at $t_i = 0$ we have $W(t_i) = W_i$.

Notice that the interval $1 \leq Y(t) \leq \infty$ is mapped bijectively into the interval $1 \leq W(t) \leq \sqrt{3}$ where $Y(t) = 1$ is mapped to the logarithmic branch point at $W(t) = 1$ while $Y(t) = \infty$ becomes $W(t) = \sqrt{3}$. Again the evolution of the ghost cosmology starts off at some value $Y_i > 1$ at initial time $t_i = 0$ and condenses towards $Y(t_c) \rightarrow 1$ at later times. Since $Y(t_c) = 1$ gives a logarithmic singularity in the solution, we see that the condensation process again takes an infinite amount of time, i.e. we have to set $t_c = \infty$. Moreover, we have to choose the solution with positive \tilde{m} to have a time-flow in positive direction. If the initial condition is such that $Y_i \gg 1$ the solution becomes

$$|\tilde{m}|t = 2 \ln \left| \left(\frac{\sqrt{3} - 1}{W(t) - 1} \right) \left(\frac{W(t) + 1}{\sqrt{3} + 1} \right) \right| - W(t) + \sqrt{3}. \quad (79)$$

D. $d < 1$

Finally for the last case with $d < 1$ one finds

$$|\tilde{m}|t = \frac{1}{\sqrt{1-d}} \left(\arctan \left(\frac{R(t)}{\sqrt{1-d}} \right) - \arctan \left(\frac{R_i}{\sqrt{1-d}} \right) \right) + \frac{2}{\sqrt{d}} \ln \left| \frac{V(t)}{V_i} \right|, \quad (80)$$

(sending $|\tilde{m}|t \rightarrow -|\tilde{m}|t$ also gives a valid solution, however with opposite time arrow) where we have defined

$$R(t) = \frac{Y(t) + 1 - d}{\sqrt{3Y^2(t) - 2Y(t) - 1 + d}}. \quad (81)$$

and initial values R_i, V_i at $t_i = 0$. It is easy to check that $R(Y(t))$ is a monotonically decreasing function along the interval $1 \leq Y < \infty$ covering the range

$$\frac{2-d}{\sqrt{d}} \geq R(Y(t)) > \frac{1}{\sqrt{3}}. \quad (82)$$

Therefore, the time t is monotonically decreasing with Y . Thus, with time running forwards we have to evolve from large to small Y values. At the condensation point, $Y(t_c) = 1$, we have $V(t_c) = \infty$ from which we see that once again the condensation process of the ghost will take an infinite amount of time, hence $t_c = \infty$.

E. Transition from Radiation to Dark Energy Dominated Universe

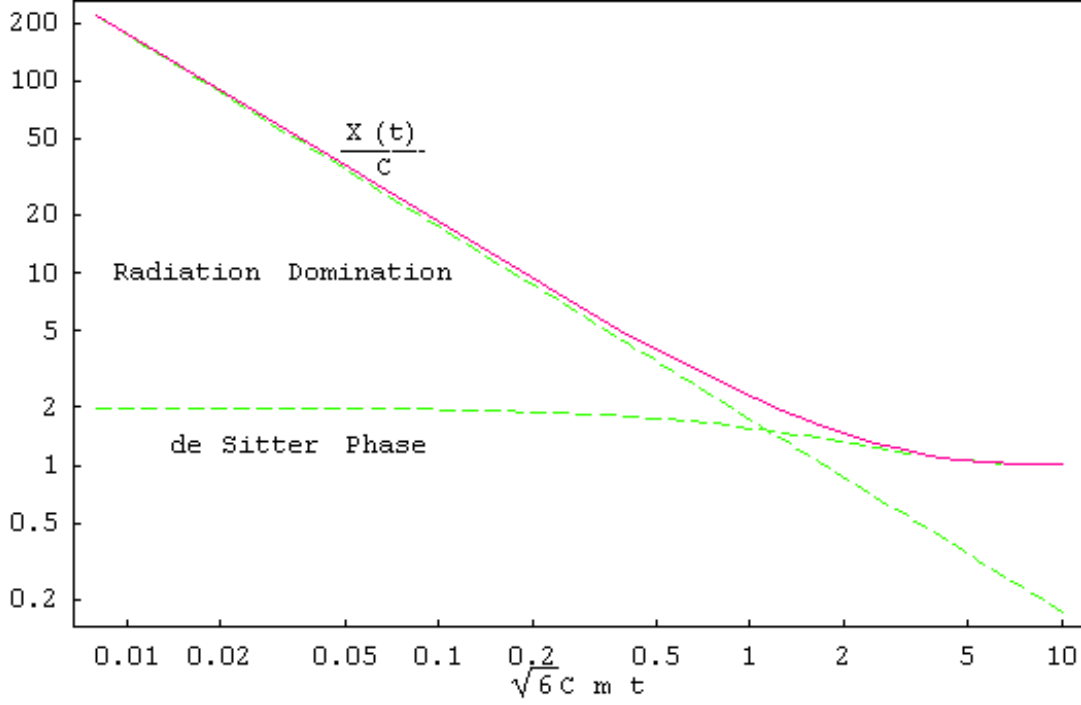


FIG. 3: Plot of the ghost field $X(t)$ (upper red curve) as a function of time for a ghost cosmology with $P(X) = \frac{1}{2}(X - C)^2 - D$; $D = \frac{2}{3}C^2$. One can clearly see the transition from an early radiation dominated universe (steep green dashed line) to a late time (dark) vacuum energy dominated universe (flat green dashed line). The transition occurs during $\sqrt{6}Cmt \sim 0.1 - 5$.

To make these cosmological solutions more transparent, let us look at their behavior at early and late times. By expanding the solutions around $Y(t_i) = Y_i \gg 1$ at early times $t \rightarrow t_i = 0$ one finds for all the different cases that

$$X(t) = \frac{M_{Pl}}{\sqrt{2}M^2 t} \quad (83)$$

$$a(t) \propto \left(\frac{M^2}{M_{Pl}} t \right)^{1/2}. \quad (84)$$

On the other hand at the late stages of the evolution upon condensation of the ghost field where $Y(t) \rightarrow 1$ and $t \rightarrow t_c = \infty$ one finds for all cases the general behavior

$$X(t) = C \left(1 + e^{-\sqrt{3D} \frac{M^2}{M_{Pl}} t} \right) \quad (85)$$

$$a(t) \propto e^{\sqrt{\frac{D}{3}} \frac{M^2}{M_{Pl}} t}. \quad (86)$$

Here we have expressed these general results in the original parameters. Hence all the obtained solutions describe irrespective of the value of D a *transition from an early radiation dominated epoch to a dark energy dominated de Sitter cosmology at late epoch*. At the same time this means a transition from deceleration to acceleration as stated at the beginning of this section.

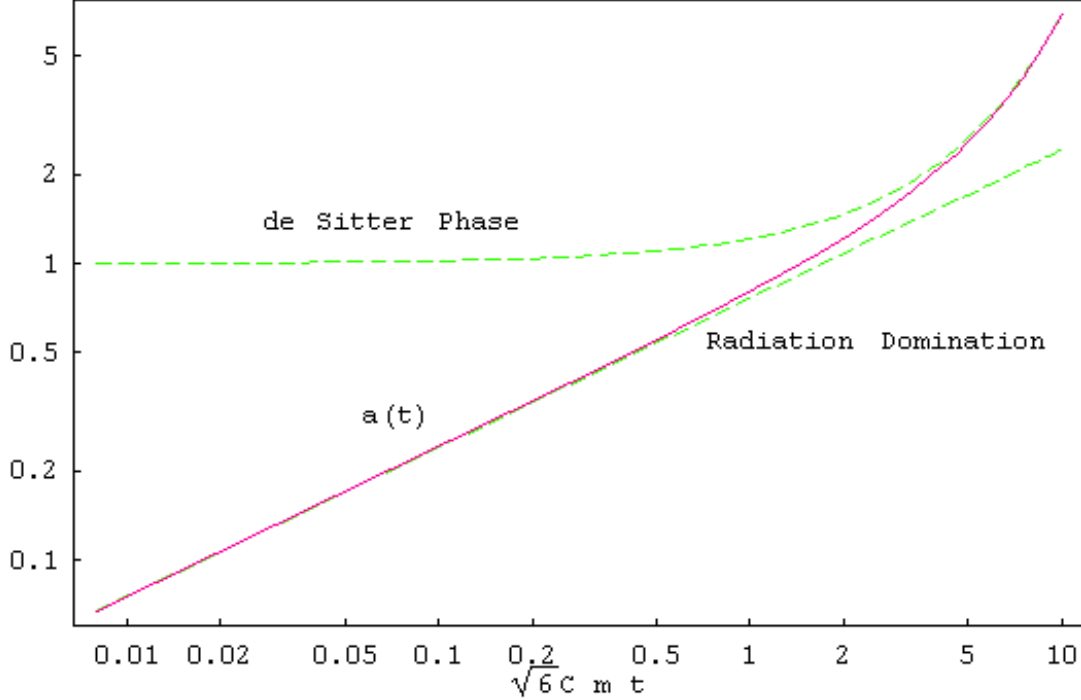


FIG. 4: Plot of the scale-factor $a(t)$ (red middle full curve) as a function of time for a ghost cosmology based on $P(X) = \frac{1}{2}(X - C)^2 - D$; $D = \frac{2}{3}C^2$. It interpolates during the time interval $\sqrt{6}Cmt \sim 0.2 - 7$ between a $t^{1/2}$ behavior (radiation dominated, lower green dashed curve) at early times and a $\exp(\sqrt{D/3}mt)$ behavior (vacuum energy dominated, upper green dashed curve) at late times.

Again the behavior of these limiting cases can be quickly understood by inspection of the density and pressure associated with the ghost field and the related equation of state. From (28) we obtain that at early times

$$\rho_\phi \rightarrow 3p_\phi, \quad (87)$$

while at late times

$$\rho_\phi \rightarrow -p_\phi. \quad (88)$$

These are indeed the right equations of state showing that the ghost field behaves like radiation at an early stage and like vacuum energy at a late stage.

Let us now look in more detail at the complete transition described by the given full solutions. Since all these solutions show a very similar dependence, we will pick for concreteness the easiest case of $d = 4/3$ and choose its initial value $Y_i \gg 1$ which leads to a late time (dark) vacuum energy, $M^4 D = \frac{2}{3} M^4 C^2$. The explicit transition from the radiation dominated to the vacuum energy dominated phase is depicted in Fig.3 for $Y(t)$ and in Fig.4

for the scale-factor $a(t)$. One observes that the transition occurs during the time interval $\sqrt{6}Cmt = 0.1 - 5$. We will see in section VII that the current age of the universe estimated by $t_0 = 1/H_0$ leads to $\sqrt{6}Cmt_0 \sim 1$. The plots in Fig.3 and Fig.4 show that this is *right in the middle of the transition phase* - very similar to what we have found before in section III in the dark matter case.

VI. FROM LARGE TO SMALL COSMOLOGICAL CONSTANT?

Both theories with quadratic function $P(X)$ which we have studied in section III and V enjoyed a radiation dominated behavior with characteristic $a(t) \sim t^{1/2}$ at early times. Moreover we have seen in section IV that with a higher power $P(X)$ we can also generate more general but exotic power-law expansions at an early epoch. In view of the inflationary phase expected to have happened in our universe at very early times, it will be also interesting to ask whether such an inflationary phase could be described by a suitable $P(X)$. Moreover, in view of the current astronomical evidence for dark energy [9], one would like to join it smoothly after various intermediate epochs again to a late time accelerated de Sitter type expansion, however this time with a drastically smaller vacuum energy. Irrespective of the intermediate epochs which might differ from a de Sitter type expansion, let us now – by assuming that the ghost cosmology framework applies way back to the times of inflation – analyze the structure of the corresponding $P(X)$. To describe a transition between a de Sitter phase with large cosmological constant and one with a small cosmological constant the requirements on $P(X)$ are:

- $P(X)$ must possess (at least) two extrema, $X = C_1$ and $X = C_2$ with $P'(C_1) = P'(C_2) = 0$ at negative $P(C_1) < P(C_2) < 0$. The latter condition guarantees a positive vacuum energy at the extremal points (to realize a very small late-time cosmological constant, one would furthermore require a suitably small vacuum energy $-P(C_2)M^4 \simeq \text{meV}^4$).

Given such a $P(X)$ we must moreover require that

- The cosmological evolution must run from C_1 – the extremal point with larger vacuum energy – to C_2 , the point with smaller vacuum energy. This guarantees a transition from a large positive cosmological constant $\Lambda_1 = -P(C_1)M^4/M_{Pl}^2$ at early times to a small cosmological constant $\Lambda_2 = -P(C_2)M^4/M_{Pl}^2$ at late times.

Two explicit classes of functions satisfying the first requirements on $P(X)$ are the cubic (with positive parameters $b, D > 0$)

$$P(X) = -(X - C)^3 + b(X - C)^2 - D \quad (89)$$

for which the critical points are

$$C_1 = C \quad \Rightarrow \quad P(C_1) = -D \quad (90)$$

$$C_2 = C + \frac{2}{3}b \quad \Rightarrow \quad P(C_2) = -D + \frac{4}{27}b^3 \quad (91)$$

and the Mexican hat kinetic function (with positive parameters $g, D > 0$)

$$P(X) = -(X - C)^2 + \frac{1}{2g^2}(X - C)^4 - D \quad (92)$$

whose two relevant critical points (the left minimum and the maximum in between the two minima) are

$$C_1 = C - g \quad \Rightarrow \quad P(C_1) = -D - \frac{1}{2}g^2 \quad (93)$$

$$C_2 = C \quad \Rightarrow \quad P(C_2) = -D. \quad (94)$$

Both critical points are such that along $[C_1, C_2]$ the slope $P'(X)$ is positive and also the remaining stability (8) and positivity constraints (22) can be satisfied along this interval. A more general $P(X)$ describing various intermediate epochs might however be much more complicated.

We will, however, argue that whatever the $P(X)$ satisfying the above first requirement might be, no transition from large to small cosmological constant is possible within the present framework (based on a 3d flat metric Ansatz (9)). In order to see this, let us start from the differential equation for X (25) and observe that $\ln |(P')^2 X|$ becomes $-\infty$ at both C_1 as well as C_2 . Hence, since the time evolution must bring us from C_1 to C_2 , the time-derivative of $\ln |(P')^2 X|$ has to start off at C_1 with a large positive value and end at C_2 with a large negative value. Therefore, the lhs of (25) has to change sign in the interval $[C_1, C_2]$. Let us now distinguish two cases. First, if $P'(X) \geq 0$ throughout the interval $[C_1, C_2]$ then $\sqrt{2XP' - \bar{P}} > 0$ on the interval and thus the rhs of (25) cannot change sign as would be required by the lhs. Thus no solution of (25) exists for the whole interval $[C_1, C_2]$. Second, in case that $P'(X) < 0$ at some point in the interval, (8) would be violated and the cosmological evolution from C_1 to C_2 would suffer instabilities against fluctuations along its way. We can therefore conclude that in both cases no consistent solution exists which would describe an evolution from C_1 to C_2 and thus a transition from large to small cosmological constant. Notice that this argument is valid for general $P(X)$ and thus cannot be avoided by introducing a bunch of intermediate epochs – as long as the whole evolution from C_1 to C_2 is dominated by the ghost.

VII. EVOLUTION OF DARK ENERGY/MATTER IN THE COMBINED GHOST - RADIATION - MATTER COSMOS

Let us in this section study the cosmological evolution in the presence of the ghost as well as additional ordinary matter and radiation described by perfect fluids. With n additional perfect fluids the Einstein equation governing the behavior of the FRW metric becomes

$$G_{\mu\nu} = 8\pi G(T_{\phi,\mu\nu} + \sum_{i=1}^n T_{i,\mu\nu}), \quad (95)$$

with $T_{\phi,\mu\nu}$ the ghost energy-momentum tensor (6) and

$$T_{i,\mu\nu} = (\rho_i + p_i)u_\mu u_\nu + p_i g_{\mu\nu} \quad (96)$$

the energy-momentum tensor for the i th perfect fluid. A fluid which is comoving in the FRW rest frame possesses a velocity $u^\mu = (1, 0, 0, 0)$ such that its energy-momentum tensor becomes simply $T_{i,\mu\nu} = (\rho_i, p_i g_{rr}, p_i g_{\theta\theta}, p_i g_{\phi\phi})$.

The Einstein equation implies the local energy-conservation of the combined ghost-fluid energy-momentum tensor. Motivated by the tiny currently observed amount of 4% ordinary

baryonic matter as opposed to 70% dark matter [9], we will assume in the sequel that the back-reaction of the perfect fluids on the geometry is negligible as compared to the back-reaction of the ghost which we will hold responsible in this section for all of the current dark matter resp. energy. That this assumption, once true at the current epoch, remains valid at earlier epochs will be shown explicitly below [17]. It is thus reasonable that the ghost and the fluids satisfy separate energy conservation laws

$$\nabla^\mu T_{\phi,\mu\nu} = 0 \ , \quad \nabla^\mu T_{i,\mu\nu} = 0 \ . \quad (97)$$

As well-known, the fluid energy conservation law in an FRW background is equivalent to

$$\dot{\rho}_i + 3H(\rho_i + p_i) = 0 \ . \quad (98)$$

Together with an equation of state

$$p_i = w_i \rho_i \ , \quad w_i = \text{const} \quad (99)$$

for the i th fluid one obtains for the mass density the solution

$$\frac{\rho_i(t)}{\rho_{i,0}} = \left(\frac{a_0}{a(t)} \right)^{3(1+w_i)} \ , \quad (100)$$

with $\rho_{i,0}, a_0$ the present values. Similarly the ghost energy conservation governs the ghost's dynamics as it is equivalent to the ghost's equation of motion (16) and is solved by (13). Of course the local energy-conservation of the total mass density is just a consequence of the Einstein equation and therefore not independent.

The Einstein equation in the FRW background provides us with two independent equations, the *Friedmann equation*

$$H^2 = \frac{1}{3M_{Pl}^2} \left(\rho_\phi + \sum_i \rho_i \right) \quad (101)$$

and the evolution equation

$$H^2 + 2\frac{\ddot{a}}{a} = -\frac{1}{M_{Pl}^2} \left(p_\phi + \sum_i p_i \right) \ . \quad (102)$$

If we eliminate H^2 in this equation with help of the Friedmann equation we arrive at the *acceleration equation*

$$\frac{\ddot{a}}{a} = -\frac{1}{6M_{Pl}^2} \left(\rho_\phi + 3p_\phi + \sum_i (\rho_i + 3p_i) \right) \ . \quad (103)$$

The formal solution to the Friedmann equation is given by

$$a(t) = a_0 e^{g(t,t_0)} \ , \quad g(t,t_0) = \pm \frac{1}{\sqrt{3}M_{Pl}} \int_{t_0}^t dt' (\rho_\phi + \sum_i \rho_i)^{1/2} \ . \quad (104)$$

Plugging this into the acceleration equation to eliminate the scale-factor leads to the equation

$$\mp \frac{2M_{Pl}}{\sqrt{3}} \frac{d}{dt} (\rho_\phi + \sum_i \rho_i)^{1/2} = (\rho_\phi + p_\phi) + \sum_i (\rho_i + p_i) \ , \quad (105)$$

which determines the evolution of the densities once the equation of states (99) are used together with (100) and (104). The resulting differential equation is no longer straightforward to solve. Fortunately, this is not needed since a potential explanation of dark matter or dark energy at the current epoch through the ghost would imply that today the ghost dominates the fluid components by far and we will shortly see that this feature also persists if we evolve the ghost-radiation-matter system back to much earlier and also later times.

Nevertheless, before proceeding with the full combined ghost-radiation-matter system let us look at two important limiting cases. The first is the limit of *early times* for the theory given by $P(X) = \frac{1}{2}(X - C)^2 - D$. At early times we know that the baryonic matter contribution is negligible as compared to radiation. The latter has an equation of state $\rho_R = 3p_R$. Moreover at these early times we have found that for this $P(X)$ the ghost also behaves like radiation with $\rho_\phi = 3p_\phi$. Therefore in this regime (105) can be easily solved by (choosing the relevant minus sign)

$$t \rightarrow 0 : \quad \rho_\phi + \rho_R = \frac{3M_{Pl}^2}{4t^2} \quad (106)$$

leading via (104) to

$$t \rightarrow 0 : \quad a(t) \propto t^{1/2} . \quad (107)$$

The second interesting limit is for *late times* for the theory defined by the kinetic function $P(X) = \frac{1}{2}(X - C)^2$. This kinetic function did not lead to dark energy at late times but instead to a non-relativistic dark matter behavior with $\rho_\phi \gg p_\phi$. Furthermore we know that at late times ordinary matter dominates over radiation such that we have here for the perfect fluid an equation of state $\rho_B \gg p_B$ as well. Also in the late time limit (105) is easy to solve through (again with the relevant minus-sign)

$$t \rightarrow t_c : \quad \rho_\phi + \rho_B = \frac{4M_{Pl}^2}{3t^2} \quad (108)$$

which via (104) leads to the corresponding late time scale-factor behavior

$$t \rightarrow t_c : \quad a(t) \propto t^{2/3} . \quad (109)$$

A. Time-Dependence of Ghost Dark Matter, Radiation and Matter

Let us now study the time evolution of ghost dark matter, baryonic matter and radiation. We will therefore take the theory with $P(X) = \frac{1}{2}(X - C)^2$ in which the ghost led to non-relativistic matter at a late epoch. We know from observation, that nowadays baryonic visible matter contributes 4% and radiation only 0.008% to the total mass density of the universe as compared to 26% coming from dark matter [9]. We will here equate the latter with the ghost dark matter. It will therefore be a good approximation at the current epoch to neglect the backreaction of ordinary baryonic matter and radiation against that of the ghost. In this ghost dominated background we will now evolve baryonic matter and radiation according to the simple power-law behavior (100) with equation of state parameters $w_B = 0$ and $w_R = \frac{1}{3}$ and scale-factor $a(t)$ given by (38). We will shortly see that the dominance of the ghost over baryonic matter and radiation becomes even more pronounced when we look back in time. Also at late times the dominance of the ghost will be preserved. This

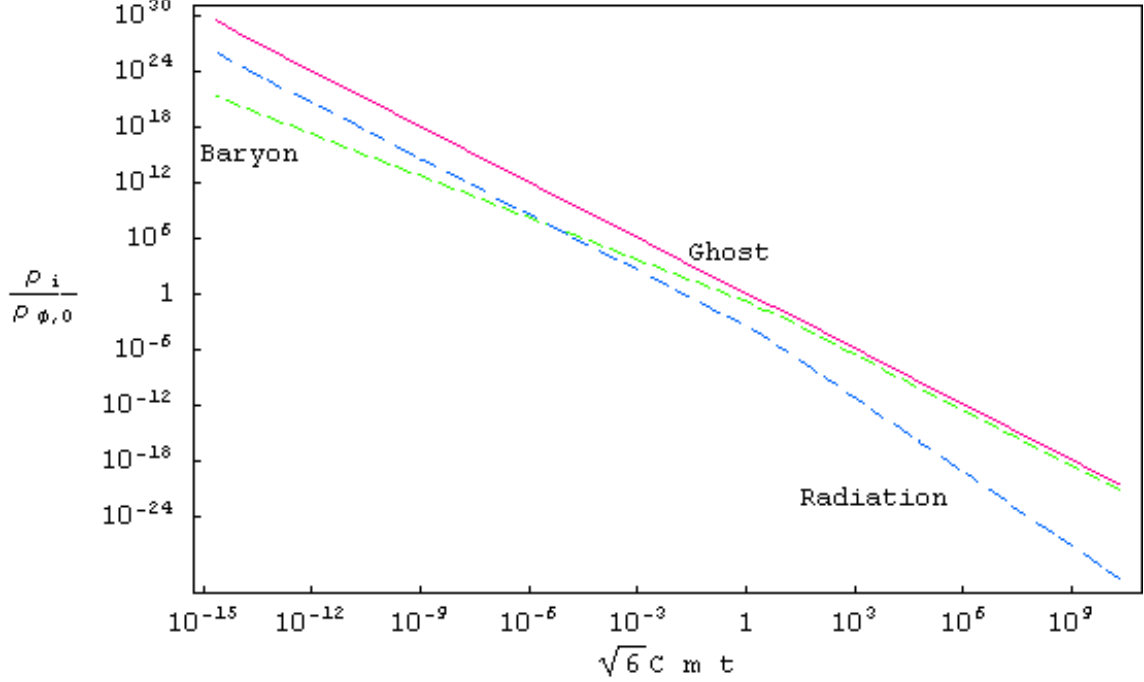


FIG. 5: Time dependence of the mass densities $\rho_\phi, \rho_B, \rho_R$ for the ghost scalar (which is assumed to account for the present dark matter), baryonic matter and radiation in a ghost cosmology based on $P(X) = \frac{1}{2}(X - C)^2$. If the ghost accounts for the dark matter today then it dominates ordinary matter and radiation not only today but even more so in the past as well.

will then establish the relevance of our cosmological solution (34), (38) which took only the backreaction of the ghost into account.

We still have to fix the relative normalizations of the densities which we will do at the current epoch. To this end, let us estimate the age of the universe by the current Hubble parameter, $t_0 = 1/H_0 \simeq 1/(10^{-33}\text{eV})$. To obtain a current dark matter density of order meV^4 we have to adjust the current ghost mass density $\rho_\phi = 2C^2 M^4 (Y_0 - 1)$ at this order (here $Y(t_0) = Y_0$). It is therefore natural (avoiding a finetuning of Y_0) to set $CM^2 \sim \text{meV}^2$. With this information we arrive at a value for $\tilde{m}t_0 = \sqrt{6}Cmt_0$ which is close to 1. Consequently $Y_0 = 2.32$ and the time-dependence of the scale-factor (38) becomes

$$\frac{a_0}{a(t)} = \frac{Y(t)^{1/6}(Y(t) - 1)^{1/3}}{1.26}. \quad (110)$$

Combining this information with the present measured densities [9], [10] (the present radiative contribution $\rho_{R,0}$ arises from the contribution of three relativistic neutrinos plus the cosmic microwave background photons [11] and the assumption of a Hubble parameter $H_0 = 100 h \text{ km sec}^{-1} \text{ Mpc}^{-1}$ with $h = 0.71$ [12])

$$\rho_{\phi,0} : \rho_{B,0} : \rho_{R,0} = 26 : 4 : 8 \times 10^{-3} \quad (111)$$

allows us to fix the relative normalization. Thus, the properly normalized mass densities

become

$$\frac{\rho_\phi}{\rho_{\phi,0}} = \frac{(Y(t) - 1)(3Y(t) + 1)}{10.27} \quad (112)$$

$$\frac{\rho_B}{\rho_{\phi,0}} = \frac{4}{26} \times \left(\frac{a_0}{a(t)} \right)^3 \quad (113)$$

$$\frac{\rho_R}{\rho_{\phi,0}} = \frac{8 \times 10^{-3}}{26} \times \left(\frac{a_0}{a(t)} \right)^4. \quad (114)$$

with $\rho_{\phi,0} \equiv \rho_{DM,0}$ the current dark matter density. Together with (110) this gives all densities as functions of $Y(t)$ and thus via the inverted solution (34) as functions of cosmic time t . The time-dependence of $\rho_\phi, \rho_B, \rho_R$ is plotted in Fig.5. We see once more that the ghost behaves like radiation at early times and assumes a non-relativistic matter-like behavior at late times. The most important aspect of Fig.5, however, is that the *ghost's domination over all other contributions persists to earlier epochs and becomes even more pronounced at earlier times*. At later epochs the dominance of the ghost will be preserved as well. Our approximation to choose (34) as the background at all times, not only at the present epoch, proves therefore to be justified and establishes that our simple cosmological solution (34), (38) is actually the relevant one also in the presence of ordinary matter or radiation.

To obtain the normalized mass densities $\Omega_\phi = \rho_\phi/\rho_c$ and $\Omega_i = \rho_i/\rho_c$, obeying $\Omega_\phi + \Omega_B + \Omega_R = 1$, one would have to divide further by the critical density $\rho_c = 3M_{Pl}^2 H^2$. Since we use the approximation that the ghost alone determines the geometry, i.e. that $\rho_c = \rho_\phi + \rho_B + \rho_R \simeq \rho_\phi$, it follows that ρ_c resp. H (given by (30)) are known functions of $Y(t)$ as well. Thus the solution (34) fully specifies the evolution of all Ω_ϕ, Ω_i . In particular in this approximation, we will have $\Omega_\phi \simeq 1$ at any time.

B. Time-Dependence of Ghost Dark Energy, Radiation and Matter

Let us next use the ghost field to account not for dark matter but instead for the full amount of dark energy currently observed. To this end, we have to take the theory with ghost kinetic function $P(X) = \frac{1}{2}(X - C)^2 - D$ leading to a late-time vacuum energy $M^4 D$ when the ghost condenses. Again, from observation, radiation and baryonic matter densities are negligible at the current epoch as compared to the 70% contribution to the mass density coming from dark energy which we equate with the current ghost mass density $\rho_{\phi,0}$. We are therefore entitled to neglect all but the backreaction of the ghost on the geometry which means that we have at present an FRW geometry with scale factor given by one of the solutions in section V. Since all of them behave similarly let us take the explicit solution (76) for $D = \frac{2}{3}C^2$ with initial condition $Y_i \gg 1$. If the ghost field accounts for all of the present dark energy, which we will assume in this subsection, we have to require that $M^4 D \sim M^4 C^2$ is of order meV^4 .

Let us be more precise about the constants appearing in the scale-factor and the mass densities. To this end we will again estimate the current age of the universe by $t_0 = 1/H_0 \simeq 1/(10^{-33}\text{eV})$ which brings $\tilde{m}t_0 = \sqrt{6}Cmt_0$ close to 1. With knowledge of t_0 we can then fix the overall constant in the scale-factor $a(t)$. Via (76) this leads to

$$\frac{a(t)}{a_0} = \frac{1}{0.80} \sqrt{1 - e^{-\sqrt{2}Cmt}} e^{\sqrt{2}Cmt/3}. \quad (115)$$

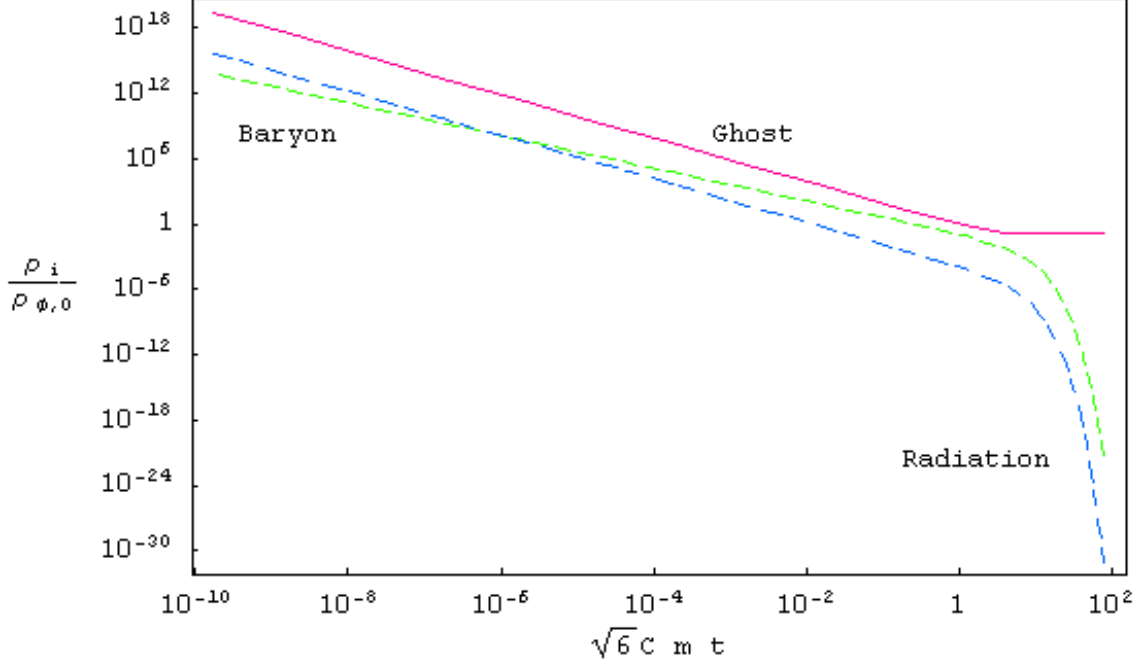


FIG. 6: Time dependence of the mass densities $\rho_\phi, \rho_B, \rho_R$ for the ghost field (which is identified here with the dark energy), baryonic matter and radiation. Close to the present age of the universe, where $\sqrt{6} C m t_0 \simeq 1$, we find that the ghost field switches from a radiative phase to a dark energy phase.

Moreover from (75) we obtain that $Y_0 = 2.28$. Next, we fix the relative normalizations of the mass densities through the current observed mass densities for dark energy (which in this subsection will be identified with the ghost mass density), baryonic matter and radiation [9]

$$\rho_{\phi,0} : \rho_{B,0} : \rho_{R,0} = 70 : 4 : 8 \times 10^{-3}. \quad (116)$$

This gives us the properly normalized mass densities ($d = 4/3$)

$$\frac{\rho_\phi}{\rho_{\phi,0}} = \frac{((Y(t) - 1)(3Y(t) + 1) + d)}{11.37} = \frac{(3Y(t) - 1)^2}{34.11} \quad (117)$$

$$\frac{\rho_B}{\rho_{\phi,0}} = \frac{4}{70} \times \left(\frac{a_0}{a(t)} \right)^3 \quad (118)$$

$$\frac{\rho_R}{\rho_{\phi,0}} = \frac{8 \times 10^{-3}}{70} \times \left(\frac{a_0}{a(t)} \right)^4. \quad (119)$$

as functions of $Y(t)$ where $\rho_{\phi,0} = \rho_{DE,0}$ is the currently observed dark energy density. With the solution for $Y(t)$ (75) we finally obtain the full time dependence for these densities which is plotted in Fig.6.

We see from Fig.6 that the hierarchy among the three contributions, which exists today if the ghost accounts for all of dark energy, extends to earlier and later epochs likewise. We therefore see that the assumption *to neglect the backreaction of radiation and matter on the FRW geometry and to take only the backreaction of the ghost into account is a valid assumption at all epochs*. Herein lies the importance of the cosmological solutions obtained

in section V. Around $\sqrt{6}Cmt_* \sim 5 \times 10^{-6}$ we reach the point where the baryonic matter and radiation densities coincide. At $t = t_*$ the ghost field has already adopted a radiative behavior as it runs with the same slope as the radiation density. At even earlier times radiation will dominate baryonic matter as in the standard hot big bang cosmology but both are still dominated by far by the ghost's mass density. Notice that while both ρ_B and ρ_R have the same dependence on the scale-factor in the dark matter case (last subsection) and the dark energy case (this subsection), nevertheless their dependence on time is different in both cases since the scale-factor $a(t)$ has a very different time-dependence in these two cases. Let us mention that for the normalized densities $\Omega_\phi, \Omega_B, \Omega_R$ the same remarks as in the preceding subsection apply. In particular we have $\Omega_\phi \simeq 1$ at all epochs.

The plot in Fig.6 reveals another interesting aspect concerning the coincidence problem. The coincidence problem arises in standard cosmology when the currently observed dark energy is explained through an ordinary cosmological constant. The matter density associated with the cosmological constant stays constant during cosmic evolution. Thus, though the cosmological constant would be the dominant energy contribution at present, this quickly changes once one goes back to earlier times. Hence, the current observation that the dark matter, the cosmological constant and to a lesser extent also the matter contribution are of the same order of magnitude present the coincidence problem. Notice that this kind of *coincidence problem is much more harmless in the ghost cosmology framework if we attribute dark energy completely to the ghost*. Though at late times the ghost acts like a cosmological constant its mass density grows towards earlier times, keeping the hierarchy among the mass densities currently observed more or less intact. Therefore, this hierarchy is not specific to a finetuned snapshot of today's universe. The real question which remains is to explain the hierarchy *at some point in time* (see e.g. [13]). The time-dependence of the ghost's mass density should have interesting cosmological consequences for the verification or falsification of the ghost gravity proposal.

C. Describing both Dark Matter and Dark Energy by the Ghost

Up to now we have considered the dark matter and dark energy cases separately by assuming that the ghost accounts for all of dark matter or all of dark energy today. This was motivated by the ghost's behavior towards late times. However, we have seen that actually at the present epoch we are right at the transition region from the early to the late time asymptotics. Therefore, the ghost cosmology based on $P(X) = \frac{1}{2}(X - C)^2 - D$ is indeed capable of describing both dark matter and dark energy at the same time as we will now discuss.

For this, let us split the ghost's mass density into a dark matter part, ρ_{DM} , and a dark energy part, ρ_{DE}

$$\rho_\phi = \rho_{DM} + \rho_{DE} \quad (120)$$

where

$$\rho_{DM} = \frac{1}{2}M^4(X - C)(3X + C), \quad \rho_{DE} = M^4D. \quad (121)$$

The associated pressures are

$$p_{DM} = \frac{1}{2}M^4(X - C)^2, \quad p_{DE} = -M^4D. \quad (122)$$

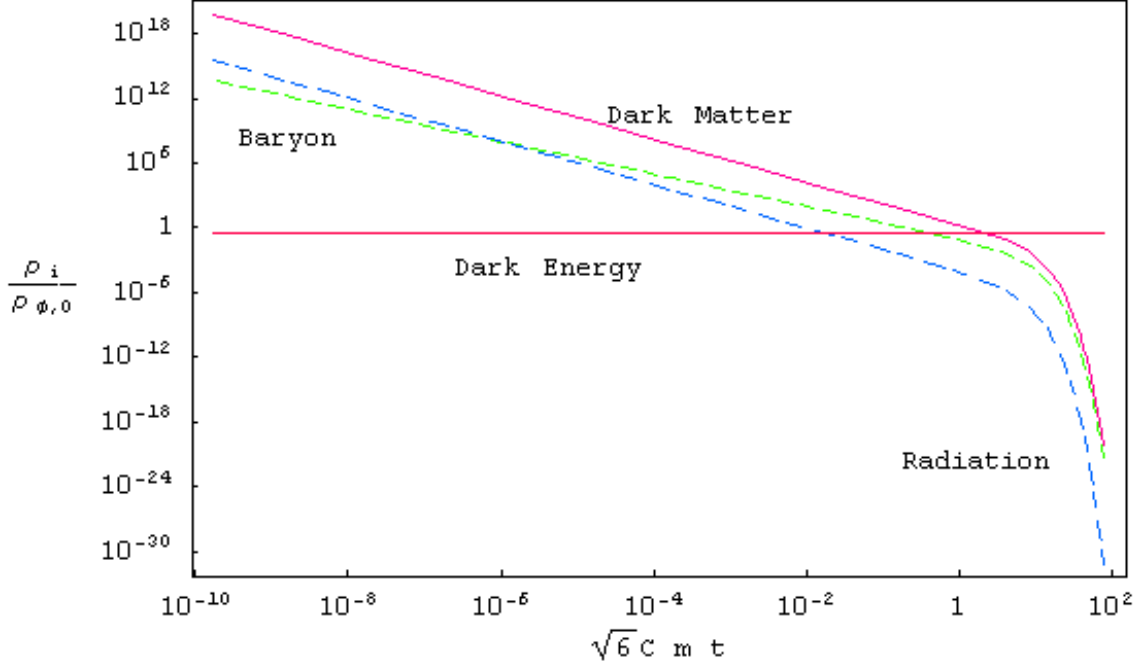


FIG. 7: Time dependence of the mass densities $\rho_{DE}, \rho_{DM}, \rho_B, \rho_R$ for the ghost field (which is identified here with both the dark energy and dark matter), baryonic matter and radiation. Close to the present age of the universe, where $\sqrt{6} C m t_0 = 1.674$, we find that the ghost field switches from a radiative phase to a dark energy phase.

Thus the dark energy component is nothing but a standard cosmological constant. Its evolution is therefore trivial and can be fixed once and for all by identifying ρ_{DE} with the current observed dark energy density $\rho_{DE,0}$. This gives a value of $Y(t_0) = 1.614$ which translates into $\tilde{m}t_0 = \sqrt{6} C m t_0 = 1.674$. Notice that we are using a different method of arriving at the $Y(t_0)$ and $\tilde{m}t_0$ as we have an additional quantity to fit. We find that this value of $\tilde{m}t_0$ coincides with our previous estimates which have shown that this value is of order one. On the other hand, the dark matter component is the same as in subsection A, only that the normalization is slightly different. The splitting that we have done to match the current dark matter and dark energy, though an important step in obtaining a realistic cosmology, leads us back to the coincidence problem for the constant ρ_{DE} .

VIII. JEANS-INSTABILITY

Let us finally analyze the stability of the obtained ghost cosmological solutions in section III and V under gravitational collapse, i.e. Jeans instability. A universe with mass density ρ is unstable under perturbations if the perturbations have a wavelength λ greater than the critical Jeans wave length

$$\lambda_J = \frac{v_s}{\sqrt{G\rho}}. \quad (123)$$

We neglect here factors of order one on the rhs which depend on the detailed geometrical properties of the perturbed equilibrium state and the perturbations. The isothermal speed

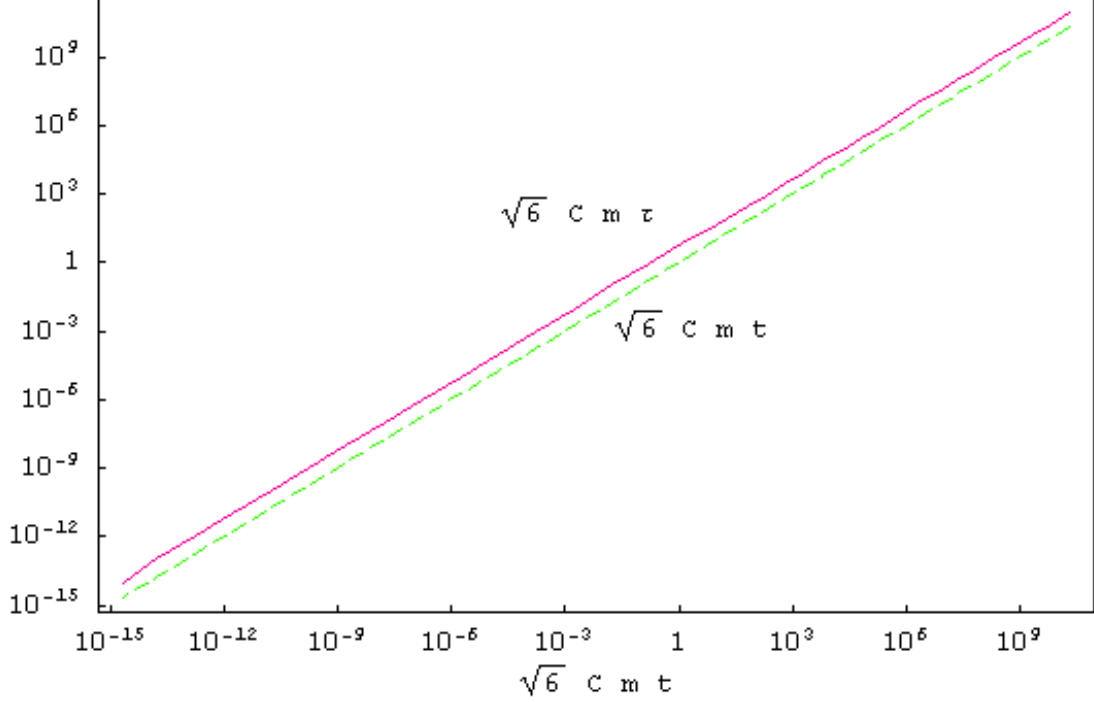


FIG. 8: Plot of cosmic time t and of the free-fall time τ as functions of time for the ghost cosmology based on $P(X) = \frac{1}{2}(X - C)^2$. At all times we have $\tau > t$ indicating that no Jeans instability arises in the corresponding cosmological solutions of section III.

of sound is denoted by v_s . The corresponding time scale measuring the total time of the ensuing collapse if $\lambda > \lambda_J$ is given by the free-fall time

$$\tau = \frac{1}{\sqrt{G\rho}} = \frac{\sqrt{8\pi}M_{Pl}}{\sqrt{\rho}}. \quad (124)$$

Thus, as long as the cosmic time t satisfies $t < \tau$, there was not sufficient time for the universe to have collapsed and it will be stable. To the contrary, for $t > \tau$ the universe had enough time to collapse and will suffer from a Jeans instability. For the ghost cosmologies based on $P(X) = \frac{1}{2}(X - C)^2 - D$ we have to set $\rho = \rho_\phi$ with

$$\rho_\phi = M^4 \left(\frac{1}{2}(X - C)(3X + C) + D \right) = \frac{C^2 M^4}{2} \left((Y - 1)(3Y + 1) + d \right). \quad (125)$$

Let us first examine the dark matter case, i.e. the theory with $D = 0$. The comparison between t (from 34) and the free-fall time τ is presented in Fig.8. We see that in this case no Jeans instability arises at any time. To understand this better let us investigate the early and late time limits. For the ghost mass density these are

$$\rho_\phi \rightarrow \begin{cases} t \rightarrow 0 & : \quad \frac{3}{2}M^4 X^2 \\ t \rightarrow \infty & : \quad 2CM^4(X - C) \end{cases} \quad (126)$$

This leads to the following expressions for the free-fall time

$$\tau \rightarrow \begin{cases} t \rightarrow 0 & : \quad \frac{4\sqrt{\pi}}{\sqrt{3}mX} \\ t \rightarrow \infty & : \quad \frac{2\sqrt{\pi}}{m(X-C)} \end{cases} \quad (127)$$

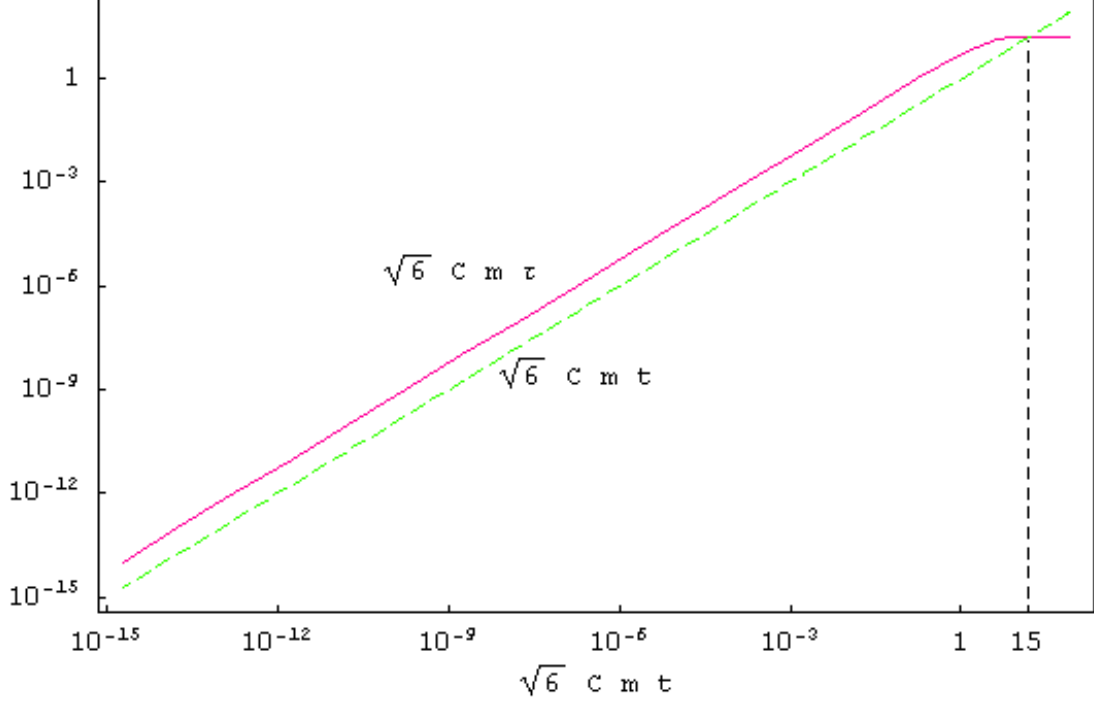


FIG. 9: Plot of cosmic time t and of the free-fall time τ as functions of time for the ghost cosmology based on $P(X) = \frac{1}{2}(X - C)^2 - D$, ($D = \frac{2}{3}C^2$ is used for the plot). At early times we find $\tau > t$ indicating that no Jeans instability arises. However at late times we see that there is a crossover to a regime with $\tau < t$. Hence a Jeans instability occurs at late times for the cosmological solutions of section V.

On the other hand, the cosmic time in these limits can be found from the expressions in section III as

$$t \rightarrow \begin{cases} t \rightarrow 0 & : \frac{1}{\sqrt{2mX}} \\ t \rightarrow \infty & : \frac{\sqrt{2}}{\sqrt{3Cm}\sqrt{X-C}} \end{cases} \quad (128)$$

Obviously in both limits, we find that $\tau > t$. Notice that in the late time limit the ghost condenses, $X \rightarrow C$.

For the dark energy case with nonvanishing $D \neq 0$ we see, however, from the plot in Fig.9 that a Jeans instability arises at late times (the actual time when this happens depends on D but is always in the future, i.e. later than t_0). Let us again look at the limiting expressions for this case. We find for the mass densities

$$\rho_\phi \rightarrow \begin{cases} t \rightarrow 0 & : \frac{3}{2}M^4 X^2 \\ t \rightarrow \infty & : M^4 D \end{cases} \quad (129)$$

which leads to the following free-fall times at early and late times

$$\tau \rightarrow \begin{cases} t \rightarrow 0 & : \frac{4\sqrt{\pi}}{\sqrt{3mX}} \\ t \rightarrow \infty & : \frac{\sqrt{8\pi}}{m\sqrt{D}} \end{cases} \quad (130)$$

To compare, we find for the cosmic time from the expressions in section V that

$$t \rightarrow \begin{cases} t \rightarrow 0 & : \frac{1}{\sqrt{2mX}} \\ t \rightarrow \infty & : -\frac{\ln(X-C)}{\sqrt{3Dm}} \end{cases} \quad (131)$$

Therefore, at early times we find $\tau > t$ and consequently no Jeans-instability can occur. However, at late times we clearly have $t > \tau$ due to the logarithmic divergence in t indicating a Jeans-instability at a late epoch. Notice that the precise time, $t_{crossover}$, when the Jeans instability will set in depends on D . E.g. in the case of $D = \frac{2}{3}C^2$ one finds that $\sqrt{6}Cmt_{crossover} = 15.04$. Since for $D \rightarrow 0$ we must recover the previously analyzed case which had no Jeans-instability at any time, we recognize that sending $D \rightarrow 0$ implies sending the crossover time, at which $\tau = t_{crossover}$, towards infinity. Thus for small enough D , the Jeans instability can be pushed far into the future. Notice that the Hubble friction which is present when the instability sets in, will cure it.

Additionally, higher derivative terms will not affect the stability. At early times, the higher derivative fluctuations are suppressed with respect to $P(X)$ while at late times, $P(X)$ is vanishing and hence higher derivative fluctuations might play a part. However, the detailed fluctuation analysis done in [1] shows that we do have a stable background.

IX. CONCLUSION

We have derived the exact cosmological solutions for a universe composed primarily of a ghost condensate. It was further shown that these solutions would smoothly interpolate between different standard cosmologies. We find, generically for quadratic kinetic functions, an early-time radiation-dominated universe which transitions into a de-Sitter expansion phase or a late-time matter-dominated scenario depending on whether the cosmological constant is present or not. This could also be understood in terms of the stress energy tensor of the ghost which gives rise to an equation of state that clearly exhibits early-time radiation-like and late-time dust-like behavior or de Sitter acceleration. For higher order kinetic functions we find more exotic power law FRW universes. An important result is that, given the current age of the universe in terms of the Hubble time, we are placed right at the transition regime. Moreover, the cosmological solutions show that the ghost condensation process takes an infinite amount of time. This preserves us from entering the regime of unstable vacua in finite cosmic time.

As a first step towards constructing fully realistic cosmologies, we considered the evolution of the universe by extrapolating from present dark energy/matter, baryonic and radiation densities to earlier times. We find that the ghost contribution completely dominates at all times and hence must be the principal driving force of cosmological evolution. Once we adjust for the dark matter/energy densities observed today by assuming that the ghost can account for these, our solutions map out the entire evolution of these densities. The other important consideration is the Jeans instability and we have shown that for the ghost dark matter case no such instability arises. In contrast, the dark energy case exhibits a late-time Jeans instability. It occurs when the universe has already entered the de Sitter expansion phase which means that Hubble friction should cure the instability.

Acknowledgments

The authors are grateful to Markus A. Luty and Lorenzo Sorbo for interesting discussions. We have been supported by the National Science Foundation under Grant Number PHY-0354401. During the final stages of this project, S.P.N. was supported by the Department Of Energy under contract DE-FG02-91ER40626.

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- [1] N. Arkani-Hamed, H. C. Cheng, M. A. Luty and S. Mukohyama, “Ghost condensation and a consistent infrared modification of gravity,” JHEP **0405**, 074 (2004) [arXiv:hep-th/0312099].
 - [2] M. Fierz and W. Pauli, “On Relativistic Wave Equations for Particles of Arbitrary Spin in an Electromagnetic Field,” Proc. Roy. Soc. Lond. A **173**, 211 (1939).
C. Brans and R.H. Dicke, “Mach’s Principle and a Relativistic Theory of Gravitation”, Phys. Rev. **124**, 925 (1961).
P.G. Bergmann, “Comments on the Scalar Tensor Theory”, Int. J. Theor. Phys. **1**, 25 (1968).
G.R. Dvali, G. Gabadadze and M. Porrati, “4D Gravity on a Brane in 5D Minkowski Space”, Phys. Lett. B **485**, 208 (2000) [arXiv:hep-th/0005016].
T. Jacobson and D. Mattingly, “Gravity with a Dynamical Preferred Frame”, Phys. Rev. D **64**, 024028 (2001).
K. Freese and M. Lewis, “Cardassian Expansion: A Model in which the Universe is Flat, Matter Dominated, and Accelerating”, Phys. Lett. B **540**, 1 (2002).
M.E. Soussa and R.P. Woodard, “A Nonlocal Metric Formulation of MOND”, Class. Quant. Grav. **20**, 2737 (2003) [arXiv:astro-ph/0302030].
C. Wetterich, “Cosmology with Varying Scales and Couplings”, arXiv:hep-ph/0302116.
J. Khoury and A. Weltman, “Chameleon Fields: Awaiting Surprises for Tests of Gravity in Space”, arXiv:astro-ph/0309300.
 - [3] S.W. Hawking and G.F. Ellis, “The Large Scale Structure of Spacetime”, Cambridge University Press, 1973.
 - [4] S. Kachru, R. Kallosh, A. Linde and S. Trivedi, “De Sitter Vacua in String Theory”, Phys. Rev. D **68**, 046005 (2003), [arXiv:hep-th/0301240].
M. Becker, G. Curio and A. Krause, “De Sitter Vacua from Heterotic M-Theory”, Nucl. Phys. B **693**, 223 (2004), [arXiv:hep-th/0403027].
A. Krause, “On De Sitter Vacua in Strongly Coupled Heterotic String Theory”, arXiv:hep-th/0404001.
 - [5] D. Krotov, C. Rebbi, V. Rubakov and V. Zakharov, “Holes in the ghost condensate,” arXiv:hep-ph/0407081.
N. Arkani-Hamed, H. C. Cheng, M. Luty and J. Thaler, “Universal dynamics of spontaneous Lorentz violation and a new spin-dependent inverse-square law force,” arXiv:hep-ph/0407034.
V. V. Kiselev, “Ghost condensate model of flat rotation curves,” arXiv:gr-qc/0406086.
L. Senatore, “Tilted ghost inflation,” arXiv:astro-ph/0406187.
F. Piazza and S. Tsujikawa, “Dilatonic ghost condensate as dark energy,” JCAP **0407**, 004 (2004) [arXiv:hep-th/0405054].
A. V. Frolov, “Accretion of ghost condensate by black holes,” arXiv:hep-th/0404216.
B. Holdom, “Accelerated expansion and the Goldstone ghost,” JHEP **0407**, 063 (2004) [arXiv:hep-th/0404109].

- M. Peloso and L. Sorbo, “Moving sources in a ghost condensate,” arXiv:hep-th/0404005.
S. L. Dubovsky, “Star tracks in the ghost condensate,” JCAP **0407**, 009 (2004) [arXiv:hep-ph/0403308].
R. J. Scherrer, “Purely kinetic k-essence as unified dark matter,” Phys. Rev. Lett. **93**, 011301 (2004) [arXiv:astro-ph/0402316].
B. M. Gripaios, “Modified gravity via spontaneous symmetry breaking,” arXiv:hep-th/0408127.
- [6] M.A. Luty, private communication.
 - [7] N. Arkani-Hamed, P. Creminelli, S. Mukohyama and M. Zaldarriaga, “Ghost Inflation,” JCAP **0404**, 001 (2004) [arXiv:hep-th/0312100].
 - [8] M. Gell-Mann, P. Ramond and R. Slansky, in “Supergravity”, eds. P. van Nieuwenhuizen and D. Freedman, North Holland, 1979.
T. Yanagida, in “Workshop on the Unified Theory and Baryon Number in the Universe”, eds. O. Sawada and A. Sugamoto, KEK, 1979.
S.L. Glashow, in “Quarks and Leptons”, eds. M. Levy *et al.*, Cargèse, 1979.
R.N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. **44**, 912 (1980).
 - [9] D.N. Spergel *et al.*, Astrophys. J. Suppl. **148**, 175 (2003) [arXiv:astro-ph/0302209].
 - [10] L. Verde, [arXiv:astro-ph/0306272].
 - [11] P.J.E. Peebles, “Principles of Physical Cosmology”, Princeton University Press, 1993.
 - [12] J.R. Mould *et al.*, Astrophys. Journ. **529**, 698 (2000) [arXiv:astro-ph/9909269].
W.L. Freedman, “Determination of Cosmological Parameters”, arXiv:astro-ph/9905222.
 - [13] A. Krause, “A Small Cosmological Constant, Grand Unification and Warped Geometry”, arXiv:hep-th/0006226.
A. Krause, “A Small Cosmological Constant and Backreaction of Non-Finetuned Parameters”, JHEP **0309**, 016 (2003), [arXiv:hep-th/0007233].
 - [14] R. M. Wald, “General Relativity”, University of Chicago Press, 1984.
 - [15] Opposite to [1] we are using in this paper a metric signature $(-, +, \dots, +)$ and the general relativity conventions of [3].
 - [16] Again we follow the conventions of [3].
 - [17] It was pointed out by L. Sorbo that the conventional Big Bang nucleosynthesis requires that non-standard relativistic matter in the radiation era to be no more than 10%. If indeed the conventional Big Bang nucleosynthesis scenario holds, this would be a serious constraint on the ghost condensate proposal. However, the ghost to ordinary matter coupling, which is outside the scope of our present paper, could significantly alter the nucleosynthesis mechanism thereby circumventing this constraint.